

Tavola dei limiti notevoli

Limiti per $x \rightarrow 0$ di $f : \mathbb{R} \rightarrow \mathbb{R}$:

$$\begin{aligned} \lim_{x \rightarrow 0} (1+x)^{\frac{1}{x}} &= e & (1) \\ \lim_{x \rightarrow 0} \frac{(1+x)^\alpha - 1}{x} &= \alpha \quad \alpha \neq 0 & (2) \\ \lim_{x \rightarrow 0^+} x^x &= 1 & (3) \\ \lim_{x \rightarrow 0^+} x^{\frac{1}{x}} &= 0 & (4) \\ \lim_{x \rightarrow 0} \frac{e^x - 1}{x} &= 1 & (5) \\ \lim_{x \rightarrow 0} \frac{a^x - 1}{x} &= \log a & (6) \end{aligned}$$

Funzioni logaritmiche:

$$\begin{aligned} \lim_{x \rightarrow 0^+} x^b \log x &= 0 \quad \forall b > 0 & (7) \\ \lim_{x \rightarrow 0} \frac{\log(1+x)}{x} &= 1 & (8) \\ \lim_{x \rightarrow 0} \frac{\log_a(1+x)}{x} &= \frac{1}{\log a} & (9) \end{aligned}$$

Funzioni trigonometriche:

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{\sin x}{x} &= 1 & (10) \\ \lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} &= \frac{1}{2} & (11) \\ \lim_{x \rightarrow 0} \frac{1 - \cos x}{x} &= 0 & (12) \\ \lim_{x \rightarrow 0} \frac{\tan x}{x} &= 1 & (13) \\ \lim_{x \rightarrow 0} \frac{\arcsin x}{x} &= 1 & (14) \\ \lim_{x \rightarrow 0} \frac{\arctan x}{x} &= 1 & (15) \end{aligned}$$

Limiti per $x \rightarrow \infty$ di $f : \mathbb{R} \rightarrow \mathbb{R}$:

$$\begin{aligned} \lim_{x \rightarrow \pm\infty} \left(1 + \frac{1}{x}\right)^x &= e & (16) \\ \lim_{x \rightarrow +\infty} \frac{a^x}{x^b} &= +\infty \quad \forall a > 1, b > 0 & (17) \\ \lim_{x \rightarrow -\infty} a^x |x|^b &= 0 \quad \forall a > 1, b > 0 & (18) \\ \lim_{x \rightarrow +\infty} \sqrt[x]{x} &= \lim_{x \rightarrow +\infty} x^{\frac{1}{x}} = 1 & (19) \\ \lim_{x \rightarrow \infty} \frac{e^x}{x^b} &= \infty & (20) \\ \lim_{x \rightarrow +\infty} \frac{\log x}{x^b} &= 0 \quad \forall b > 0 & (21) \\ \lim_{x \rightarrow +\infty} \frac{\log x}{e^x} &= 0 & (22) \end{aligned}$$

Forme indeterminate:

$$(8) \quad \frac{0}{0}, \quad \frac{\infty}{\infty}, \quad 0 \cdot \infty, \quad 1^\infty, \quad 0^0, \quad (\pm\infty)^0, \quad +\infty - \infty.$$

(9) Confronto di infiniti e infinitesimi:

Se

$$\lim_{n \rightarrow \infty} |a_n| = \infty$$

Allora:

$$(11) \quad \log_a n \leq n^b \leq c^n \leq n! \leq n^n \quad \text{con } a, b, c > 1$$

(12) Se

$$\lim_{n \rightarrow \infty} |a_n| = 0$$

Allora:

$$(14) \quad \frac{1}{\log_a n} \geq \frac{1}{n^b} \geq \frac{1}{c^n} \geq \frac{1}{n!} \geq \frac{1}{n^n} \quad \text{con } a, b, c > 1$$

(15)